

Theory of confinement-induced interlayer molecular resonances

M. Kanász-Nagy¹, E. A. Demler² and G. Zaránd³

¹*Condensed Matter Research Group of the Hungarian Academy of Sciences, Budafoki út 8., H-1111 Budapest, Hungary*

²*Department of Physics, Harvard University, Cambridge, MA 02138, U.S.A and*

³*BME-MTA Exotic Quantum Phases Research Group, Budapest University of Technology and Economics, Budapest 1521, Hungary*

We study theoretically the interaction between two species of bosons confined parabolically in one transverse direction but with a finite separation between the centers of the confining potentials. We demonstrate the existence of new types of confinement-induced Feshbach resonances that can be tuned by changing separation between the layers. They are much sharper than usual two-dimensional confinement-induced resonances, and can be observed in the shaking spectrum of the two-dimensional layers, as shown by detailed many-body calculations for a thermal Bose gas.

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The rich world of low-dimensional quantum systems has attracted and continues to attract immense interest. Indeed, restriction of the particles' motion to lower dimensions increases the role of both interactions and quantum fluctuations, and gives rise to exotic quantum states such as Luttinger liquids, fractional quantum Hall [1] and quantum spin Hall states [2, 3] as well as various spin liquid states, not to mention the family of high temperature superconductors, where effective two-dimensionality seems to play a determining role [4].

Ultracold atoms opened radically new perspectives in the study of low-dimensional quantum systems. By means of deep optical lattices [5, 6], single 'pancake' and 'cigar-shaped' traps [7, 8] or Hermite-Gaussian laser beams [9] one can now create quasi-two-dimensional and one-dimensional structures at ease, and can study their dynamical and interaction properties systematically [5–8, 10–13]. In fact, increasing precision and experimental control gradually promoted these optical systems to 'quantum simulators' [14, 15], allowing to test the validity of various theoretical approaches systematically. These optical systems also go beyond their condensed matter counterparts in that they allow the creation of *multicomponent* fermionic and bosonic systems in restricted geometries while controlling their interaction, and thereby open the possibility to realizing new quantum states of matter [16–19], never observed before.

Controlling the interaction between various species in these restricted geometries is of primary importance. As we know from the seminal works of Olshanii [20] and Petrov and Shlyapnikov [21, 22], confinement substantially changes the interaction in quasi-one and two dimensions [23]. Confinement-tuning has indeed been used recently to control the interaction within one-dimensional Bose gases [6, 24–26].

In quasi-two dimensions, in particular, it leads to the emergence of a bound state of energy E_b for all values of the three-dimensional scattering length, a_{3D} , thereby making the interaction of low-energy quasiparticles repulsive, and leads to the emergence of a logarith-

mically broad scattering resonance at collision energies $\epsilon \sim E_b$. The existence of these bound states has indeed been observed both in quasi-one and in quasi-two dimensions [8, 27–31].

While our theoretical understanding of the interactions of one-component quasi-one- and quasi-two-dimensional bosons is quite complete, double-layer systems are much less explored. The purpose of the present work is to study how separation between the layers influences the interaction between the species and the fate of quasi-bound states. As we show below, the structure of the effective interactions changes radically as one separates the two layers; new resonances identified as interspecies quasi-bound states appear at previously forbidden transition energies. Moreover, in contrast to the very broad resonances identified by Petrov and Shlyapnikov, these resonances become much sharper as one separates the layers and, as we demonstrate through the computation of the 'shaking spectrum' of a double-layer thermal gas, they should be clearly observable in experiments. In addition, upon changing the separation between the two layers, new Feshbach-like resonances emerge in the low energy scattering amplitude, thereby allowing a geometrical control of the effective interlayer interaction. The geometrically-tuned interaction control opens a route to realizing novel interaction-driven quantum phases of layered two-dimensional multicomponent systems. In condensed matter, such systems are of great interest due to their exotic behavior: excitonic Bose condensates [32, 33] and exotic integer and fractional quantum-Hall states [34–39] have been predicted and observed in bilayer two-dimensional quantum well structures. More recently, a similarly rich quantum Hall behavior [40, 41] and the emergence of zero field magnetic phases has been discovered in bilayer graphene [42–44]. The special structure of topological excitations in certain two-dimensional multicomponent systems is also expected to modify the character of phase transition [45, 46].

We shall study bosonic atoms of two different hyperfine states, labeled by $\alpha = \uparrow$ and \downarrow , confined optically along

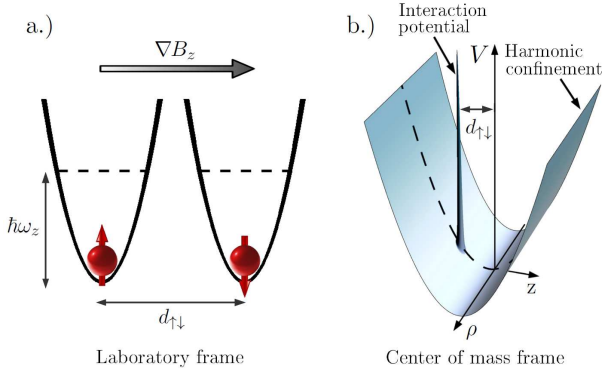


FIG. 1. (Color online.) Left: sketch of the experimental setup. A two component gas is optically confined to quasi-two dimensions, and the two layers are separated using strong magnetic field gradients. Right: Interaction and harmonic trapping potential in relative coordinates of two particles of opposite spins.

the z direction with potential minima at positions z_{\uparrow}^0 and z_{\downarrow}^0 , respectively, and interacting through the short-ranged potentials, $g_{\alpha\alpha'}\delta(\mathbf{r}-\mathbf{r}')$, characterized by the corresponding three-dimensional scattering lengths, $a_{\uparrow\uparrow}$, $a_{\uparrow\downarrow}$ and $a_{\downarrow\downarrow}$. We assume a parabolic confinement, described by the single particle Hamiltonians

$$\mathcal{H}_{\uparrow,\downarrow} = \frac{\mathbf{p}^2}{2m} + \frac{m\omega_z^2}{2} z_{\uparrow,\downarrow}^2, \quad (1)$$

with the z coordinates $z_{\uparrow,\downarrow} = z - z_{\uparrow,\downarrow}^0$ measured from the centers of the layers [47], and the separation $d_{\uparrow\downarrow} \equiv z_{\downarrow}^0 - z_{\uparrow}^0$ of the two layers being tunable through the application of an external magnetic field gradient (see Fig. 1). Individual particle states in the two layers can then be classified by their momentum \mathbf{q} within the xy plane and a harmonic oscillator quantum number n , referring to their motion along the z direction.

We start our analysis by studying the scattering of two particles on each other. Clearly, Eq. (1) as well as the interaction allow us to separate center of mass (Z) and relative coordinates ($z \equiv z_{\alpha} - z_{\beta}$). In particular, in the absence of interactions, the two particles' wave function in the z direction can be expressed as $\Psi \sim \varphi_N(\sqrt{2Z})\varphi_{\nu}(z/\sqrt{2})$, with φ_{ν} denoting the usual harmonic oscillator wave functions for an oscillator length $l_z \equiv \sqrt{\hbar/(m\omega_z)}$, and N and ν the center of mass and relative motions' quantum numbers, respectively. In the center of mass frame, particles of opposite spin interact through the off-center interaction in Fig. 1.b. Two incoming particles of opposite momenta $q = |\mathbf{q}|$ and of energy $\epsilon = \hbar^2 q^2/m + \nu\hbar\omega_z$, can scatter from channel ν into channel ν' , provided that the outgoing channel is 'open', $\nu'\hbar\omega_z \leq \epsilon$. A lengthy but relatively straightforward generalization of the computations of Ref. [22] to our case yields the following expression for the scattering

amplitudes in the open channels,

$$f_{\alpha\beta}^{\nu\nu'}(\epsilon) = \frac{4\pi a_{\alpha\beta} \phi_{\nu}(d_{\alpha\beta})\phi_{\nu'}(d_{\alpha\beta})}{1 + \frac{a_{\alpha\beta}}{\sqrt{2\pi} l_z} w_{\alpha\beta}(\epsilon/\hbar\omega_z)}. \quad (2)$$

Here $d_{\alpha\beta} \equiv z_{\beta}^0 - z_{\alpha}^0$ stands for the separation between layer α and β , $\phi_{\nu}(z)$ denotes $\phi_{\nu}(z) \equiv \varphi_{\nu}(z/\sqrt{2})/\sqrt[4]{2}$ and the function $w_{\alpha\beta}(\epsilon/\hbar\omega_z)$ is given by the expression

$$w_{\alpha\beta}(x) \approx - \sum_{\nu=0}^{2\bar{\nu}-1} \sqrt{2\pi} l_z \phi_{\nu}^2(d_{\alpha\beta}) \ln\left(\frac{\nu-x}{2} - i0^+\right) + c_{\bar{\nu}},$$

with $c_{\bar{\nu}} = 2\sqrt{\frac{\pi}{2}} \ln \frac{\bar{\nu}}{2}$, and $\bar{\nu} \gg \epsilon/\hbar\omega_z$ a sufficiently large cut-off parameter [48, 49]. The logarithmic terms in the expression of $w_{\alpha\beta}$ incorporate the effects of virtual transitions to open or closed transverse channels. The prefactors $\phi_{\nu}^2(d_{\alpha\beta})$ appear since the two particles must stay at the same position to interact through the potential in Fig. 1.b.

While for scattering within the same spin channels we have $d_{\uparrow\uparrow} = d_{\downarrow\downarrow} = 0$, and Eq. (2) reduces to the expression of Ref. [22], for the spin $\uparrow\downarrow$ channel $f_{\uparrow\downarrow}(\epsilon)$ depends sensitively on the distance $d_{\uparrow\downarrow}$ between the two layers through the relative wave functions, ϕ_{ν} . For small energies, $\epsilon \approx 0$ in particular, $w_{\alpha\beta}(x)$ behaves as $w_{\alpha\beta}(x) \approx e^{-d_{\alpha\beta}^2/2l_z^2} (-\ln|x| + i\pi\theta(x) + \gamma_{\alpha\beta})$, with the separation dependent dimensionless prefactor, $\gamma_{\alpha\beta}$, of order 1. Thus, the scattering amplitudes have a pole at $\epsilon < 0$, corresponding to a bound state at an energy

$$E_{\alpha\beta}^0 \propto -\hbar\omega_z e^{-1/|a_{\alpha\beta}|\phi_0^2(d_{\alpha\beta})}, \quad (3)$$

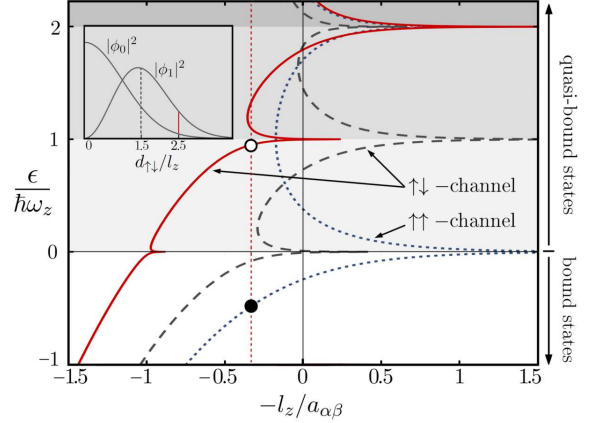


FIG. 2. (Color online.) Energies of bound and quasi-bound states in the $\uparrow\uparrow$ (dotted line) and in the $\uparrow\downarrow$ -channels (dashed and full lines) at a separation $d_{\uparrow\downarrow}/l_z = 1.5$ and 2.5 , respectively, for zero center of mass motion ($N = 0$ and $\mathbf{Q} = 0$). For $N \neq 0$ the spectrum is shifted by $N\hbar\omega_z$. The open and closed circles denote the quasi-bound and bound molecular resonances shown in Fig. 5, respectively. Inset: amplitudes $\phi_0^2(d_{\uparrow\downarrow})$ and $\phi_1^2(d_{\uparrow\downarrow})$ as functions of $d_{\uparrow\downarrow}/l_z$, determining the lifetime of the molecules as well as the strengths of logarithmic singularities of the (quasi)-bound state energies close to $\epsilon = 0$ and $\hbar\omega_z$.

for small negative scattering lengths. For positive energies, $\epsilon > 0$, the imaginary part of $w_{\alpha\beta}$ is finite, and $f_{\alpha\beta}(\epsilon)$ displays resonances of finite width each time the real part of the denominator of Eq. (2) crosses zero, indicating quasi-bound states of finite lifetime. The corresponding energies are displayed in Fig. 2 for some typical confinement parameters as a function of $l_z/a_{\alpha\beta}$. The interlayer scattering (solid line) displays features completely absent in intralayer scattering (dashed lines). While for intralayer scattering $\nu = 0 \rightarrow \nu = 1$ relative quantum number transitions are forbidden by reflection symmetry, such interlayer processes are allowed once $d_{\uparrow\downarrow} \neq 0$, and they amount in the emergence of a quasi-bound molecular state (resonance) at an energy

$$(E_{\uparrow\downarrow}^1 - \hbar\omega_z) \propto -\hbar\omega_z e^{-1/|a_{\uparrow\downarrow}|\phi_1^2(d_{\uparrow\downarrow})}, \quad (4)$$

and a corresponding resonance in the scattering amplitudes in (2). Importantly, while the binding energy of this quasi-bound state is determined by $\phi_1^2(d_{\uparrow\downarrow})$, its decay rate is proportional to the imaginary part of $w_{\uparrow\downarrow}$ at the resonance frequency, which is proportional to $\phi_0^2(d_{\uparrow\downarrow})$. Therefore, increasing the separation between the two layers of atoms, one can make the quasi-bound state sharper and sharper — at the cost of somewhat decreasing the binding energy (see also the inset of Fig. 2). Similar interlayer quasi-bound states of energy $E_{\uparrow\downarrow}^\nu$ appear close to every threshold, $\epsilon \approx \nu \hbar\omega_z$, and can turn to a narrow resonance as one increases the layer separation $d_{\uparrow\downarrow}$.

We should emphasize that Fig. 2 displays only the relative energy ϵ of the bound states and quasi-bound states in the center of mass frame. The total energy of two particles with total incoming momentum $\mathbf{Q} = \mathbf{q}_1 + \mathbf{q}_2$ is, however, given as a sum of their relative motion's energy, ϵ , and the energy associated with the center of mass motion, $E_{\text{COM}} = N\hbar\omega_z + Q^2/4m$. Accordingly, the bound state spectrum in Fig. 2 gets shifted by an energy $N\hbar\omega_z$ for each center of mass quantum number N , corresponding to excited molecular bound states with an oscillating center of mass coordinate along the z direction.

The emergent interlayer quasi-bound states can also be used to engineer the interaction between atoms in the two layers. In particular, the effective interaction in a degenerate two-dimensional Bose gas is approximately proportional to the scattering amplitude at the corresponding energy $\epsilon = 2\mu$, with the chemical potential μ [22]. Fig. 3 shows the $\uparrow\downarrow$ scattering amplitude as a function of layer separation for fixed energies $0 < \epsilon \ll \omega_z$. For the parameters chosen, the interaction initially decreases with increasing separation, and a sharp Feshbach-resonance-like structure emerges as the quasi-bound state reaches the energy of the incoming particles, leading to a very strong interaction between the two species. Crossing this resonance, the effective interaction turns from repulsive to attractive. Notice that, in contrast to single layer systems [22], these resonances appear on *both* sides of the

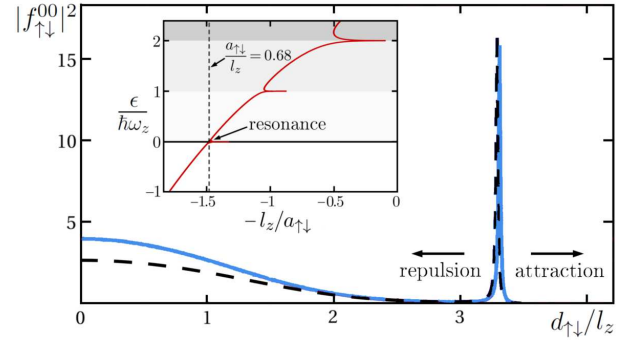


FIG. 3. (Color online.) Scattering amplitude $f_{\uparrow\downarrow}^{00}$ computed for a three-dimensional scattering length $a_{\uparrow\downarrow} = 0.68 l_z$ as a function of the layer separation, $d_{\uparrow\downarrow}$. The continuous and dashed curves correspond to energies $\epsilon/\hbar\omega_z = 0.05$ and 0.01 , respectively. A sharp Feshbach-resonance structure emerges at $d_{\uparrow\downarrow}/l_z \approx 3.3$, when the energy of incoming particles become resonant with a long-lived quasi-bound molecular state, as shown in the Inset.

three-dimensional Feshbach resonance, and, in a somewhat unusual way, they become the sharpest on the repulsive side, $a_{\uparrow\downarrow} > 0$.

The lifetime of the interlayer quasi-bound states becomes exponentially long upon increasing the separation of the layers, and they can therefore be detected in a shaking experiment. To demonstrate this, we study a strongly confined dilute thermal gas (with a temperature $T \ll \omega_z$), coupled to a time dependent magnetic field gradient that shakes the layers in opposite directions. To account for many-body effects, we describe the gas in terms of the second quantized Hamiltonian,

$$H = \int d^3\mathbf{r} \left\{ \sum_{\alpha} \psi_{\alpha}^{\dagger}(\mathbf{r}) (\mathcal{H}_{\alpha} - \mu_{\alpha}) \psi_{\alpha}(\mathbf{r}) + \sum_{\alpha,\beta} \frac{g_{\alpha\beta}}{2} \psi_{\alpha}^{\dagger}(\mathbf{r}) \psi_{\beta}^{\dagger}(\mathbf{r}) \psi_{\beta}(\mathbf{r}) \psi_{\alpha}(\mathbf{r}) \right\}, \quad (5)$$

with the chemical potentials $\mu_{\alpha} < 0$ setting the densities of the gases. The fields ψ_{α} are expanded in terms of the harmonic oscillator wave functions as $\psi_{\alpha}(\mathbf{r}) \propto \sum_{n,\mathbf{q}} \varphi_n(z - z_{\alpha}^0) e^{i\mathbf{q}\cdot\boldsymbol{\rho}} a_{\alpha n}(\mathbf{q})$, with ρ denoting the in-plane coordinates of the particles, and $a_{\alpha n}(\mathbf{q})$ their annihilation operators. Shaking is described by the modulation of the Hamiltonian $\mathcal{H}_{\uparrow,\downarrow}$,

$$\delta\mathcal{H}_{\alpha}(t) = -h_{\alpha} \cos(\omega t) z_{\alpha}/l_z, \quad (6)$$

with the fields $h_{\alpha} \sim \mu_B(\nabla\delta B)$ characterizing the amplitudes of shaking for the two hyperfine components. Clearly, shaking leads to intralayer transitions, i.e. $n = 0 \rightarrow 1$ transitions within each layer. Decomposed in relative and center of mass coordinates, however, these excitations correspond to pair excitations with quantum numbers $(N, \nu) = (1, 0)$ and $(N, \nu) = (0, 1)$. Therefore, shaking should not only allow to excite thermal particles to higher intrawell bands (at energy $\hbar\omega_z$), but —

through the interaction with other thermal bosons – also allow to excite the $\uparrow\downarrow$ -quasi-bound state of energy $E_{\uparrow\downarrow}^1$ in the $(N, \nu) = (0, 1)$ channel, and the $\uparrow\uparrow$ (and $\downarrow\downarrow$) - bound states of energy $E_{\uparrow\uparrow}^0 + \hbar\omega_z$ (or $E_{\downarrow\downarrow}^0 + \hbar\omega_z$) in the $(N, \nu) = (1, 0)$ channel [50]. We therefore expect peaks at all these energies in the absorption spectrum [51].

To determine the shaking spectrum, we employed field theoretical methods, and calculated the imaginary part of the shaking susceptibility $\chi''_\alpha(\omega)$ by neglecting vertex corrections and computing the ‘dressed’ bubble diagram shown in Fig. 4.c. The quantity $\hbar^2\omega\chi''_\alpha(\omega)$ is then directly proportional to the rate of energy absorption in layer α . Interactions between the bosons have been incorporated through the self-energies $\Sigma_\alpha^{nn'}(\omega)$ of the retarded propagators $(G_R)_\alpha^{nn'}(\omega)$,

$$(G_R^{-1})_\alpha^{nn'}(\omega, \mathbf{q}) = \omega + i0^+ + \frac{\hbar q^2}{2m} + n\omega_z \delta_{nn'} + \frac{1}{\hbar} \Sigma_\alpha^{nn'}(\omega, \mathbf{q}),$$

with the self-energies computed within the T -matrix approximation by summing up the complete ladder series for the T -matrix (vertex function), as shown in Fig. 4.a, through solving the corresponding Bethe-Salpeter equations. Details of this calculations are given in [48], here we just summarize the main results.

Fig. 5 displays the numerically computed shaking spectrum for some typical parameters in the cross-over regime, $l_z/a_{\alpha\beta} \sim d/l_z \sim 1$. For simplicity, we assumed shaking fields $h_\uparrow = h_\downarrow$, and repulsive three-dimensional scattering lengths of equal size in all three scattering channels, $a_{\alpha\beta} = a$. We can clearly distinguish three peaks in the spectra. The largest peak at $\omega \approx \omega_z$ corresponds the direct subband transitions within the same layer, and has an amplitude directly proportional to the boson density. We also observe, however, two smaller

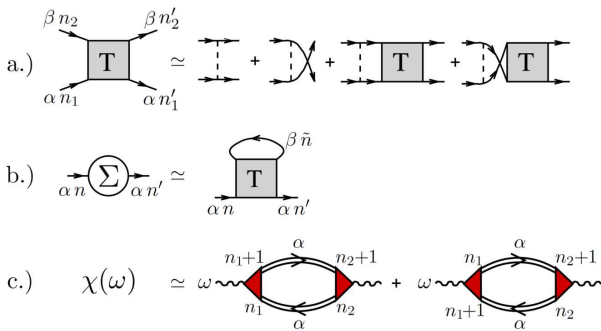


FIG. 4. (Color online.) Feynman diagrams determining the shaking spectrum: (a) Bethe-Salpeter equations of the many-body T -matrix, approximated as a sum up ladder diagrams. Full lines indicate bare propagators, dashed lines refer to the bare coupling. (b) Self-energy corrections within the T -matrix approximation. (c) Shaking susceptibility, as calculated within linear response theory and neglecting vertex corrections. Double lines indicate propagators dressed by the self-energy corrections (b), whereas red triangles stand for shaking vertices.

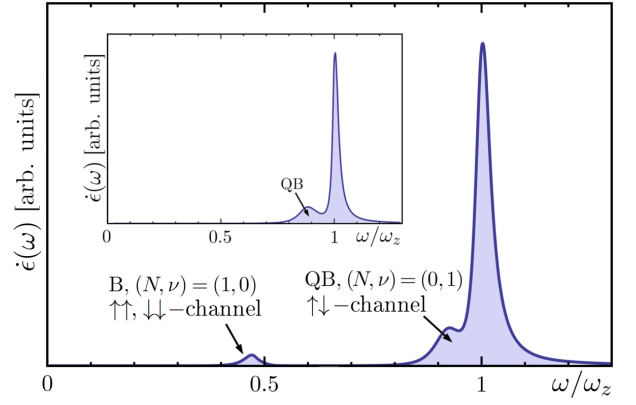


FIG. 5. (Color online.) Shaking absorption spectrum at equal interaction strengths $a_{\alpha\beta}/l_z = 3$, and layer separation $d_{\uparrow\downarrow}/l_z = 2.5$. Center of mass (N) and relative (ν) quantum numbers of the peaks related to bound (B) and quasi-bound (QB) molecular states (open and full circles in Fig. 2) are also shown. [Physical parameters: $|\mu_\uparrow| = |\mu_\downarrow| = k_B T/3$, $k_B T = 0.03 \hbar\omega_z$.] Inset: in the absence of interlayer interactions ($a_{\uparrow\uparrow} = a_{\downarrow\downarrow} = 0$), the bound states in the $\uparrow\uparrow$ and $\downarrow\downarrow$ -channels disappear. [Inset’s parameters: $a_{\uparrow\downarrow} = 2.6 l_z$, $|\mu_\uparrow| = |\mu_\downarrow| = k_B T/3$, $k_B T = 0.06 \hbar\omega_z$.]

peaks. These peaks are due to two-body processes, and therefore their intensities are proportional to the square of the boson densities. The peak next to the large quasi-particle excitation peak is due to the quasi-bound molecular state in the $\uparrow\downarrow$ channel at energy $E_{\uparrow\downarrow}^1$, indicated by the empty circle in Fig. 2. As expected, for separations $d \sim l_z$ this peak is indeed sharp enough and can be identified unambiguously within the shaking spectrum. Although this quasi-bound resonance may appear relatively weak at a first sight for the thermal gas studied here, it is expected to get more pronounced as the bosons are cooled down and the system is driven towards quantum degeneracy, a regime beyond the reach of approximations used here.

The peak at even smaller frequencies has an entirely different origin. This peak can be identified as being due to a transition into the $\uparrow\uparrow$ or $\downarrow\downarrow$ intralayer bound states combined with a center of mass excitation, $N = 0 \rightarrow 1$ (full circle in Fig. 2). Notice that – to leading order – a direct transition to the bound state is forbidden by symmetry (parity), and therefore excitation of a center of mass oscillation is necessary to observe the $E_{\uparrow\uparrow}^0$ and $E_{\downarrow\downarrow}^0$ bound states. This peak is expected to split up for $a_{\uparrow\uparrow} \neq a_{\downarrow\downarrow}$, and vanishes if we set $a_{\uparrow\uparrow}, a_{\downarrow\downarrow} \rightarrow 0$ (see inset of Fig. 5). The bound state in the $\uparrow\downarrow$ scattering gives a tiny contribution for these parameters, and is practically not visible in Fig. 5.

In summary, we have studied the scattering of particles in a bilayer quasi-two-dimensional Bose gas and have shown that the interlayer scattering amplitudes depend sensitively on the separation of the layers due to novel

confinement-induced bound and quasi-bound molecular states. This makes it possible to tune interlayer interactions in a wide range, by simple geometric means. The increased lifetime of the quasi-bound interlayer molecules makes it possible to observe them in the excitation spectrum of a shaking experiment. Dalibard

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 - [47] Shifting the z coordinate also shifts the argument of the interaction in the z direction $\delta(z-z') = \delta(z_\alpha - z_{\alpha'} - d_{\alpha\alpha'})$.
 - [48] Supplementary Information.
 - [49] Throughout this paper we assume that a set of real harmonic oscillator wave functions is used.
 - [50] The latter state becomes also a resonance once center of mass and relative coordinates are coupled [29].
 - [51] Notice that Fig. 2 displays only the relative motion's energy, and to obtain the total energy, one should add the center of mass motion's energy, $N\hbar\omega_z + Q^2/4m$.
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SUPPLEMENTARY INFORMATION

Separation of the center of mass coordinates

The interaction potential $g_{\alpha\beta} \delta(\mathbf{r}_1 - \mathbf{r}_2)$ of particles of hyperfine spin α and β depends only on their relative position, therefore their center of mass motion decouples completely. Indeed, in the center of mass and relative coordinates

$$\begin{aligned} z &= z_1 - z_2 + d_{\alpha\beta}, & Z &= \frac{z_1 + z_2 - (z_\alpha^0 + z_\beta^0)}{2}, \\ \rho &= \rho_1 - \rho_2, & \mathbf{R} &= \frac{\rho_1 + \rho_2}{2}, \end{aligned}$$

the two-particle Hamiltonian takes the simple form, $H = H_{\text{rel}} + H_{\text{COM}}$, with

$$\mathcal{H}_{\text{rel}} = \frac{p_\rho^2 + p_z^2}{m} + \frac{m\omega_z^2}{4} z^2 + g_{\alpha\beta} \delta(\rho, z - d_{\alpha\beta}), \quad (\text{S1})$$

$$\mathcal{H}_{\text{COM}} = \frac{p_{\mathbf{R}}^2 + p_Z^2}{4m} + m\omega_z^2 Z^2. \quad (\text{S2})$$

Here \mathcal{H}_{rel} describes scattering of a particle of mass $\frac{m}{2}$ in a parabolic potential with a Dirac-delta scatterer at a distance $d_{\alpha\beta}$ from the origin, as shown in Fig.1. For particles of different hyperfine spin ($d_{\uparrow\downarrow} \neq 0$) the interaction term excites transitions between all relative harmonic oscillator levels $\nu \rightarrow \nu'$, whereas quantum numbers of the center of mass motion N remain unaffected. For particles of the same hyperfine spin ($d_{\uparrow\uparrow} = d_{\downarrow\downarrow} = 0$), however, due to reflection symmetry, ν can only change by an odd number, imposing a selection rule on the scattering process.

Two-particle scattering in vacuum

We follow Ref. [22] to describe the scattering of two particles in their relative coordinates in Eq. (S1). The two-dimensional scattering amplitudes $f_{\alpha\beta}^{\nu\nu'}$ are defined through the $\rho \rightarrow \infty$ asymptotic form of the scattering wave function of an incoming particle in channel ν ,

$$\Psi_\nu \approx \phi_\nu(z) e^{i\mathbf{q}\rho} - \sum_{\nu'} f_{\alpha\beta}^{\nu\nu'}(\epsilon) \phi_{\nu'}(z) \sqrt{\frac{i}{8\pi q_{\nu'} \rho}} e^{iq_{\nu'} \rho}, \quad (\text{S3})$$

where $\epsilon = \hbar^2 q^2 / m$ is the energy of the incoming particles, and $q_\nu = \sqrt{m(\epsilon - \nu \hbar \omega_z + i0^+) / \hbar}$.

Outside the radius of the interaction potential the wave function propagates freely. Focusing on the s-wave scattering channel, where $e^{i\mathbf{q}\rho} \approx J_0(q\rho)$, this motion can be described by the ansatz

$$\Psi_\nu(\rho, z) = \phi_\nu(z) J_0(q\rho) + A_{\alpha\beta}^\nu(\epsilon) G_\epsilon(\mathbf{r}, d_{\alpha\beta}).$$

Here $A_{\alpha\beta}^\nu$ is a proportionality constant, J_0 is a Bessel function of the first kind and G_ϵ is the retarded Green's

function of the harmonic potential, centered at the point of interaction,

$$G_\epsilon(\mathbf{r}, d_{\alpha\beta}) = \sum_{\nu=0}^{\infty} \frac{\phi_\nu^*(z) \phi_\nu(d_{\alpha\beta})}{2\pi} K_0(-iq_\nu \rho),$$

with the modified Bessel functions K_0 . Close to the point of interaction, the wave function must be proportional to a three-dimensional scattered wave

$$\psi_{3D}(\mathbf{r}) \propto 1 - \frac{a_{\alpha\beta}}{|\mathbf{r} - d_{\alpha\beta} \mathbf{e}_z|}, \quad (\text{S4})$$

whereas in the limit $\rho \rightarrow \infty$ it decays according to Eq. (S3). A straightforward comparison of the limits in Eqs. (S3, S4) leads to the scattering amplitudes shown in Eq. (2), with

$$w_{\alpha\beta} = \lim_{\rho \rightarrow 0} \left(2 \sum_{\nu=0}^{\infty} \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} K_0(-iq_\nu \rho) - \frac{\sqrt{2\pi} l_z}{\rho} \right).$$

To simplify this expression, we choose a large integer $\bar{\nu}$ and split the sum above into two parts, with $\nu < 2\bar{\nu}$ and $\nu \geq 2\bar{\nu}$. Assume that ρ is already small, so that $\kappa \equiv \sqrt{2\bar{\nu}} \rho / l_z$ is a small parameter. Then, in the first part of the sum, the Bessel functions can be approximated by their asymptotic form

$$K_0(x \rightarrow 0) \sim -\ln(x/2) - \gamma_E, \quad (\text{S5})$$

whereas in the remaining terms, $\nu \geq 2\bar{\nu}$, we can approximate K_0 's argument by $\sqrt{\nu} \rho / l_z$, and make use of the asymptotic form of the Hermite functions $\phi_{\nu \rightarrow \infty}$,

$$\frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} \sim \sqrt{\frac{2}{\pi}} \frac{\cos^2\left(\frac{d_{\alpha\beta}}{l_z} \sqrt{\nu + \frac{1}{2}} - \nu \frac{\pi}{2}\right)}{\sqrt{\nu}}.$$

As ν is varied, the \cos^2 term averages out to $1/2$, whereas, since ρ / l_z is a small parameter, K_0 's argument changes only slowly. Therefore, this sum can be approximated by an integral

$$\begin{aligned} \sum_{\nu=2\bar{\nu}}^{\infty} \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} K_0(-iq_\nu \rho) &\simeq \frac{l_z}{\rho} \sqrt{\frac{2}{\pi}} \int_{\kappa}^{\infty} dx K_0(x) \\ &\simeq \frac{l_z}{\rho} \sqrt{\frac{2}{\pi}} \left(\frac{\pi}{2} + \kappa \left(\ln \frac{\kappa}{2} + \gamma_E - 1 \right) \right), \end{aligned}$$

with $x = \sqrt{\nu} \rho / l_z$. In the last equality we made use of the formula $\int_0^\infty K_0(x) dx = \pi/2$ and of the asymptotic form of K_0 in Eq. (S5). Finally, by putting the two parts of the sum for ν together, we can take the limits $\rho \rightarrow 0$ and $\nu \rightarrow \infty$, by keeping $\kappa \rightarrow 0$, and get

$$\begin{aligned} w_{\alpha\beta} &= \lim_{\bar{\nu} \rightarrow \infty} \left[c_{\bar{\nu}} - \sum_{\nu=0}^{2\bar{\nu}-1} \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} \ln \left(\frac{\nu}{2} - \frac{\epsilon + i0^+}{\hbar \omega_z} \right) \right. \\ &\quad \left. + \left(\ln(\rho / \sqrt{2} l_z) + \gamma_E \right) \left(4 \sqrt{\frac{\bar{\nu}}{\pi}} - 2 \sum_{\nu=0}^{2\bar{\nu}-1} \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} \right) \right], \quad (\text{S6}) \end{aligned}$$

where $c_\nu = 2\sqrt{\frac{\nu}{\pi}} \ln \frac{\nu}{e^2}$. The term in the second line above disappears in the $\bar{\nu} \rightarrow \infty$ limit, and we get back the desired form for $w_{\alpha\beta}$, as given below Eq. (2).

Integral formula for $w_{\alpha\beta}$

Expression Eq. (S6) has particularly slow $\mathcal{O}(\ln \bar{\nu}/\sqrt{\bar{\nu}})$ convergence, and strong oscillations in the $d_{\uparrow\downarrow} \neq 0$ case, making it impractical for numerical investigations. Therefore, generalizing the calculations of Ref. [23], we derive an integral formula for $w_{\alpha\beta}$ more suitable for numerical implementations. First, we rewrite the terms in Eq. (S5) in an integral form,

$$\frac{\sqrt{2\pi} l_z}{\rho} = \frac{1}{\sqrt{2}} \int_0^\infty \frac{d\tau}{\tau^{3/2}} e^{-\rho^2/(4l_z^2 \tau)} \quad (\text{S7})$$

and

$$\begin{aligned} \frac{1}{2\pi} K_0(-iq_\nu \rho) &= -\frac{\hbar^2}{m} \int \frac{d^2 k}{(2\pi)^2} \frac{e^{i\mathbf{k}\rho}}{\epsilon + i0^+ - \left(\frac{\hbar^2 q^2}{m} + \hbar\nu\omega_z\right)} \\ &= \int_0^\infty \frac{d\tau}{4\pi\tau} e^{\tau(\epsilon/\hbar\omega_z - \nu)} e^{-\frac{\rho^2}{4l_z^2 \tau}}, \end{aligned} \quad (\text{S8})$$

for any $\nu > \epsilon/\hbar\omega_z$. By choosing an arbitrary $\bar{\nu} > \epsilon/\hbar\omega_z$ we can rewrite Eq. (S5) in the form

$$\begin{aligned} w_{\alpha\beta} &= \lim_{\rho \rightarrow 0} \left\{ 2 \sum_{\nu=0}^{\bar{\nu}} \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} K_0(-iq_\nu \rho) \right. \\ &\quad + \int_0^\infty \frac{d\tau}{\tau} e^{-\rho^2/(4l_z^2 \tau)} \left[-\frac{1}{\sqrt{2\tau}} \right. \\ &\quad \left. \left. + \sum_{\nu=\bar{\nu}+1}^\infty \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} e^{\tau(\frac{\epsilon}{\hbar\omega_z} - \nu)} \right] \right\}. \end{aligned} \quad (\text{S9})$$

The infinite sum above can be carried out exactly by making use of the formula for the real space density matrix of a harmonic oscillator [52],

$$\sum_{\nu=0}^\infty \frac{|\phi_\nu(z)|^2}{|\phi_0(0)|^2} e^{-\tau\nu} = \sqrt{\frac{e^\tau}{2 \sinh \tau}} e^{-\tanh(\tau/2) z^2/2l_z^2}.$$

In order to take the $\rho \rightarrow 0$ limit of Eq. (S9), we expand the Bessel function up to $\mathcal{O}(\rho^2)$ order, and rewrite its $\ln(\rho)$ singularity in an integral form,

$$\begin{aligned} K_0(-iq_\nu \rho) &\approx \left[i\pi - \gamma_E + \ln 4 - \ln \left(\frac{\epsilon + i0^+}{\hbar\omega_z} - \nu \right) \right. \\ &\quad \left. + \int_0^\infty \frac{d\tau}{\tau} \Theta \left(\frac{1}{4} - \tau \right) e^{-\rho^2/(4l_z^2 \tau)} \right] / 2, \end{aligned}$$

with the Heaviside function $\Theta(\tau)$ and Euler's constant $\gamma_E \approx 0.577$. Finally, in the $\rho \rightarrow 0$ limit, $w_{\alpha\beta}$ can be

expressed as

$$\begin{aligned} w_{\alpha\beta} &= \sum_{\nu=0}^{\bar{\nu}} \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} \left(i\pi - \ln \left(\nu - \frac{\epsilon + i0^+}{\hbar\omega_z} \right) + \ln 4 - \gamma_E \right) \\ &\quad + \int_0^\infty \frac{d\tau}{\tau} \left[e^{-\frac{\tau\epsilon}{\hbar\omega_z}} \sqrt{\frac{e^\tau}{2 \sinh \tau}} e^{-\tanh(\tau/2) d_{\alpha\beta}^2/2l_z^2} - \frac{1}{\sqrt{2\tau}} \right. \\ &\quad \left. + \sum_{\nu=0}^{\bar{\nu}} \frac{|\phi_\nu(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} \left(\Theta(1/4 - \tau) - e^{\tau(\epsilon/\hbar\omega_z - \nu)} \right) \right]. \end{aligned}$$

Many-body Hamiltonian

In order to separate the motional degrees of freedom parallel and perpendicular to the two-dimensional planes in Eq. (5), we expand the fields

$$\psi_\alpha(\mathbf{r}) = \sum_{n=0}^\infty \int \frac{d^2 q}{(2\pi)^2} e^{i\mathbf{q}\rho} \varphi_n(z - z_\alpha^0) a_{\alpha n}(\mathbf{q}),$$

where $\varphi_n(z)$ are the Hermite functions in the laboratory frame. In this basis, the many-body Hamiltonian $H = H_{\text{kin}} + H_{\text{int}}$ reads

$$\begin{aligned} H_{\text{kin}} &= \sum_{\alpha=\uparrow,\downarrow} \sum_{n=0}^\infty \int \frac{d^2 q}{(2\pi)^2} \xi_{\alpha n \mathbf{q}} a_{\alpha n}^\dagger(\mathbf{q}) a_{\alpha n}(\mathbf{q}), \\ H_{\text{int}} &= \sum_{\alpha,\beta=\uparrow,\downarrow} \sum_{\mathbf{n},\mathbf{n}'} \int \frac{d^2 k}{(2\pi)^2} \frac{d^2 k'}{(2\pi)^2} \frac{d^2 q}{(2\pi)^2} \frac{t_{\alpha\beta}^{\mathbf{n}\mathbf{n}'}}{2} \\ &\quad a_{\alpha n_1}^\dagger(\mathbf{k} + \mathbf{q}) a_{\beta n_2}^\dagger(\mathbf{k}' - \mathbf{q}) a_{\beta n_2'}(\mathbf{k}') a_{\alpha n_1'}(\mathbf{k}), \end{aligned} \quad (\text{S10})$$

where $\xi_{\alpha n \mathbf{q}} = \hbar^2 q^2/2m + n\hbar\omega_z - \mu_\alpha$, $\mathbf{n} = (n_1, n_2)$, and

$$\begin{aligned} t_{\alpha\beta}^{\mathbf{n}\mathbf{n}'} &= g_{\alpha\beta} \int_{-\infty}^\infty dz_1 dz_2 \delta(z_1 - z_2 + d_{\alpha\beta}) \\ &\quad \varphi_{n_1}^*(z_1) \varphi_{n_2}^*(z_2) \varphi_{n_2'}(z_2) \varphi_{n_1'}(z_1) \\ &= g_{\alpha\beta} \sum_{N\nu\nu'} C_{N\nu}^{\mathbf{n}*} C_{N\nu'}^{\mathbf{n}'} \phi_\nu^*(d_{\alpha\beta}) \phi_{\nu'}(d_{\alpha\beta}) \end{aligned}$$

is the bare T -matrix (vertex) of interactions. The $C_{N\nu}^{\mathbf{n}}$ above denote the overlaps $C_{N\nu}^{\mathbf{n}} = \langle N\nu | \mathbf{n} \rangle$, and the normalization of the operators in Eq. (S10) is determined by the commutation relations $[a_{\alpha n}(\mathbf{q}), a_{\alpha' n'}^\dagger(\mathbf{q}')] = (2\pi)^2 \delta_{\alpha\alpha'} \delta_{nn'} \delta(\mathbf{q} - \mathbf{q}')$.

Shaking experiment

Assuming an equal coupling of the magnetic field gradient to the spin components $h_\uparrow = h_\downarrow$, shaking of the layers is described by the modulation Hamiltonian $\delta H = -h_0 \cos(\omega t) \Xi$ in Eq. (6), with the operator

$$\Xi = \sum_{\alpha=\uparrow,\downarrow} \sum_{n=0}^\infty \int \frac{d^2 q}{(2\pi)^2} \sqrt{\frac{n+1}{2}} \left(a_{\alpha n+1}^\dagger(\mathbf{q}) a_{\alpha n}(\mathbf{q}) + \text{h.c.} \right).$$

Up to first order, shaking creates $n \leftrightarrow n+1$ transitions between harmonic oscillator levels, thus, for a thermal Bose gas in a strongly confining potential, $k_B T \ll \hbar\omega_z$, the only relevant transitions will be those between levels $n = 0 \leftrightarrow 1$. The energy absorption rate of the gas is given in linear response theory by

$$\dot{\epsilon}(\omega) = \frac{\hbar_0^2}{2} \omega \text{Im}\chi(\omega), \quad (\text{S11})$$

where the susceptibility is given by the Kubo formula

$$\chi(t) = i\Theta(t) \langle [\Xi(t), \Xi(0)] \rangle. \quad (\text{S12})$$

We approximate the susceptibility by taking into account the bubble diagrams in FIG.4c with dressed propagators and neglecting vertex corrections to the bubbles. The self-energy corrections to the propagator are due to the interaction of the propagating particle with the thermal gas through the many-body T -matrix, as shown in Fig. 4.

In the diagrammatic approach, incoming particles are specified by their frequency ω_i ('energy'), momentum \mathbf{q}_i , spin α_i , and their subband label n_i . The two particle T -matrix corresponds to the vertex function within the field theoretical approach, and it is proportional to the scattering amplitudes in Eq. (2) with the total energy of the incoming bosons replaced by the sum of the frequencies of the incoming bosons, $\Omega = \omega_1 + \omega_2$ (see Ref. 23). Both Ω and the total incoming momentum $\mathbf{Q} = \mathbf{q}_1 + \mathbf{q}_2$ are conserved within the 'ladder' diagram approximation of Fig. 4.a, unlike the quantum numbers n_1 and n_2 , which are not conserved. However, similar to the two particle problem [23], it is possible to sum up the whole ladder series by transforming to center of mass and relative coordinates and corresponding quantum numbers, $\{n_1, n_2\} \rightarrow \{N, \nu\}$. The total many-body vertex in Fig. 4.a can then be expressed as

$$\mathbf{T}_{\alpha\beta}^{\mathbf{n};\mathbf{n}'}(\Omega, \mathbf{Q}) = \sum_{N, N', \nu, \nu'} C_{N\nu}^{\mathbf{n}*} C_{N'\nu'}^{\mathbf{n}'} \mathbf{T}_{\alpha\beta}^{NN'; \nu, \nu'}(\Omega, \mathbf{Q}).$$

In the limit of strong confinement, $\hbar\omega_z \gg k_B T$ where only the lowest $n = 0$ subbands are populated, the T -matrix becomes diagonal in the center of mass index,

$\mathbf{T}_{\alpha\beta}^{NN'; \nu, \nu'} \rightarrow \mathbf{T}_{\alpha\beta}^{N; \nu, \nu'}$ and can be expressed as

$$\mathbf{T}_{\alpha\beta}^{N; \nu, \nu'}(\Omega, \mathbf{Q}) = \frac{4\pi a_{\alpha\beta}}{m} \frac{\phi_\nu^*(d_{\alpha\beta}) \phi_{\nu'}(d_{\alpha\beta})}{1 + \frac{a_{\alpha\beta}}{\sqrt{2\pi l_z}} \mathcal{W}_{\alpha\beta}^N(\Omega, \mathbf{Q})}, \quad (\text{S13})$$

$$\mathcal{W}_{\alpha\beta}^N(\Omega, \mathbf{Q}) \approx w_{\alpha\beta}(\epsilon/\hbar\omega_z) + \delta_{N0} \delta w_{\alpha\beta}^{\text{th}}(\Omega, \mathbf{Q}),$$

with $\epsilon = \hbar\Omega - N\hbar\omega_z - Q^2/4m$, and the function $\mathcal{W}_{\alpha\beta}^N$ the many-body counterpart of the function $w_{\alpha\beta}$ in equation Eq. (2). The first term in \mathcal{W} is just the vacuum contribution computed earlier, while the second term accounts for many-body interactions with other thermal bosons, and is of $\mathcal{O}(\text{density})$,

$$\delta w_{\alpha\beta}^{\text{th}}(\Omega, \mathbf{Q}) = -\frac{4\pi}{m} \frac{|\phi_0(d_{\alpha\beta})|^2}{|\phi_0(0)|^2} \Pi_{\alpha\beta}^{\text{th}}(\Omega, \mathbf{Q}),$$

with

$$\Pi_{\alpha\beta}^{\text{th}}(\Omega, \mathbf{Q}) = \sum_{\gamma=\alpha, \beta} \int \frac{d^2 q}{(2\pi)^2} n_B \left(\frac{\hbar^2 (\frac{\mathbf{Q}+\mathbf{q}}{2m})^2}{2m} - \mu_\gamma \right),$$

where n_B denotes the Bose distribution function. The contributions of $\delta w_{\alpha\beta}^{\text{th}}$ to the Green's functions turn out to be numerically small, and for thermal bosons, most features observed are dominated by the vacuum scattering amplitudes given by just the vacuum term, $w_{\alpha\beta}$. Neglecting therefore the thermal corrections to the many-body T -matrix and keeping only terms proportional to $\sim (\text{density})^2$, we can express the self-energy in Fig. 4.b as

$$\Sigma_{\alpha}^{nn'}(\omega, \mathbf{q}) \approx \sum_{\beta=\uparrow, \downarrow} \sum_{\tilde{n}=0}^{\infty} \int \frac{d^2 k}{(2\pi)^2} n_B \left(\frac{\hbar^2 \mathbf{k}^2}{2m} + \tilde{n}\hbar\omega_z - \mu_\beta \right) \mathbf{T}_{\alpha\beta}^{n\tilde{n}, n'\tilde{n}} \left(\omega + \frac{\hbar k^2}{2m} + \tilde{n}\omega_z, \mathbf{k} + \mathbf{q} \right).$$

Having the self-energies at hand, we can then proceed and compute the spectral functions of the Green's functions, $\rho_{\alpha}^{nn'}(\omega, \mathbf{q}) = -\text{Im} G_{\alpha}^{Rnn'}(\omega, \mathbf{q})/\pi$, and use that to compute the imaginary part of the 'shaking susceptibility'. Assuming strong confinement ($k_B T \ll \hbar\omega_z$), only the lowest energy channels $n = 0, 1$ give contributions, and we obtain

$$\chi_{\alpha}(\omega) = \int \frac{d\tilde{\omega} d^2 \tilde{q}}{2(2\pi)^2} n_B(\tilde{\omega} - \mu_{\alpha}) \left[\rho_{\alpha}^{00}(\tilde{\omega}, \tilde{q}) \rho_{\alpha}^{11}(\tilde{\omega} + \omega, \tilde{q}) + 2 \rho_{\alpha}^{01}(\tilde{\omega}, \tilde{q}) \rho_{\alpha}^{01}(\tilde{\omega} + \omega, \tilde{q}) + \{\omega \leftrightarrow -\omega\} \right].$$